Expressing Measurements: Numbers and the SI Units

If a student is to be successful in chemistry, or for that matter any science with quantitative aspects, the student must have a clear and complete knowledge and understanding of numbers. That is, how accurately do I know the number and what does the number represent?

Measurements are the means by which scientists gather quantitative information. The beginning student generally recognizes the importance of making a careful measurement, but usually fails to recognize that great care must also be exercised in recording the measurement. Hence, it must be recognized that there are **two basic** parts to any measurement, the number and the label (or unit) associated with it. Both must be stated clearly when quoting a measurement.

NUMBERS

The numerical portion of a measurement relates the actual value obtained and the accuracy to which it is known. In your own work, you will always want to convey, exactly, the accuracy to which a value is known. This is true because, even if the exact degree of accuracy is not important to you in your work, it may be important to someone else who, at some later time, may try to use your results for a different purpose.

There are two distinct classes of numbers. They are:

- Exact numbers (known with absolute or infinite accuracy), and
- Measured numbers (known with relative or finite accuracy).

The exact numbers can be divided into two types. They are:

- Counted numbers; as the "9" in 9 baseballs, or the "3" in 3 eggs, and
- Defined numbers; as the "12" in 12 inches equal a foot, or the "16" in 16 ounces equal a pound.

Exact Numbers

There is no uncertainty in either type of exact number because it is not possible, for example, to have a fractional number of baseballs (that is, 9.1 baseballs is not possible) or anything other than exactly 16 ounces in a pound. Therefore, it must be kept in mind that the accuracy of any calculation involving exact numbers is limited only by the accuracy of those numbers which are not exact numbers, namely those numbers which are derived from measurements. One could argue that counting 9 000 000 baseballs makes that a counted number and therefore exact. However, in this case (because the number is so large), the odds are great that some error was made in the counting. Therefore, one must use some judgement in deciding the accuracy of any such number.

Defined Numbers

Conversion factors within one system of measurement are exact; they are defined numbers. (Conversion factors between different systems of measurement usually need to be compared by some measurement and, therefore, are approximate. However, exceptions do exist--when one system is defined in terms of the other system. E.g., 1 inch ≡ 25.4 millimeters, and 1 calorie ≡ 4.184 joules. Both of these conversions are now exact.) Therefore, it is easy to see why it is desirable to do all work within one system.

Numbers like $\sqrt{2}$ (an irrational number) and π (a transcendental number) fall into the category of "defined numbers" even though you can't write the number out completely. For example, while the value of π is not defined explicitly, how it is obtained (the ratio of the circumference of a circle to its diameter) is defined and the value can be obtained to whatever degree of accuracy is needed.

It must be remembered that whenever a measurement is made, there is always some degree of uncertainty in the value of the measurement, i.e., an estimate is involved. One needs to know how to treat this uncertainty in a number and in subsequent calculations. A simple method to indicate the accuracy of the measurement (and the one that we shall use) is the use of significant figures. (Also one needs to be able to express very large or very small numbers conveniently without losing any of their accuracy).

Significant Figures

Significant figures only have meaning for measured numbers or calculations that result from measured numbers. (Exact numbers have an infinite number of significant figures.) Every measured number must have a decimal point or else its accuracy is in doubt. Significant figures are just what they say they are. If the digit has any meaning it is reported.

Significant figures are digits which are known with certainty plus one additional estimated digit.

The estimate may be fairly exact or fairly crude, depending upon the instrument used to make the measurement. The uncertainty is always explicitly stated as "±" some value, except when the uncertainty is ± 1 in the last digit. Therefore, whenever you are presented with a number, which is either a measurement or derived from a measurement, and the uncertainty is not given, it is assumed to be ± 1 in the last digit (for example, "0.05 seconds" means 0.05 seconds ± 0.01 seconds).

To determine the number of significant figures in a given measured number, begin at the left-hand side of the number and move toward the right. The first non-zero digit that you encounter and all digits to the right of it are significant.

Scientific Notation

It is often very important to express numbers in scientific notation since it avoids the ambiguity in significance of zero at the end of a number. For example, how does one express the value "9 572 000 feet," when this measurement is only good to 4 significant figures? The standard form of scientific notation has a number between zero and ten followed by a power of ten (i.e. one digit to the left of the decimal point).

Therefore, "9.572×10^6 feet" has 4 significant figures with one digit to the left of the decimal point. (The exponent only serves to place the decimal point.)

What does one do if the value is "0.52000 mile," when this measurement is good to 3 significant figures?

"0.520 mile" or "5.20×10^{-1} mile."

Notice, that the first expression has a zero before the decimal point. As stated above, there should always be one digit to the left of the decimal point, even if it's a zero. This is done because, otherwise, it would be easy to overlook the decimal point in ".520".

MEASUREMENTS

When making a measurement, you should always report the best value that you can obtain. It should be reported in the clearest and most concise manner possible. This requires a systematic approach to measurement making. A simple five-step procedure works well:

1) Look at the device itself and note A: the direction of change of the numbers (do they increase up, down, left or right?) and note B: the value of each of the smallest gaps on the device (E.g., gap = 0.1 mL).

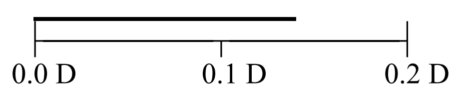
2) Write down ALL digits that are known with absolute certainty including the proper units (This will determine the number of significant figures in your final answer. Remember the definition of significant figures?). E.g., 7.4 mL.

3) Make an estimate of the fraction-of-the-smallest-spacing beyond the previous marking occupied by the object in terms of 1/10's of the gap (E.g., 3/10 of the gap). (This amount will eventually be added to the known values to reach the proper number of significant figures in the final answer.) See the example below for a discussion of the uncertainty.

4) Convert the estimated fraction into a decimal number with the proper units. This is done by multiplying the fraction of gap by the value of one gap (found in step one).
E.g., 3/10 gap × 0.1 mL/gap = 0.03 mL.

5) Add the estimated quantity to the known quantity to reach the final answer.
E.g., 7.4 mL + 0.03 mL = 7.43 mL

To illustrate the procedure, consider the following measurement:



1) The numbers on the scale increase left to right and the value of the smallest gap is 0.1 D.

2) The line is obviously longer than 0.1 D but less than 0.2 D and therefore we know the 0.1 D with absolute certainty. Consequently, the line should be reported to two significant figures. That is, by estimating the second decimal place, a total of two significant figures are obtained.

3) The next step in our measurement process is to estimate of the length of the line that extends beyond the 0.1 D marking. To do this, we divide (in our minds eye) the distance between the 0.1 D marking and the 0.2 D marking into 10 parts. (Tenths of the smallest spacing.) We can now estimate the length as 4/10 (give or take a tenth) of that distance. This is usually written as 4/10 gap ± 1/10 gap. The quantity, "± 1/10 gap", is the uncertainty in the estimate. Our "eye" may say the line segment extends 4/10 of the way past the 0.1D mark but others who look at it may see it differently, such as 3/10 or 5/10. No one will realistically see any thing other than 3/10, 4/10 or 5/10 of a gap. Thus, we must recognize that the 4/10 of a gap is merely our best "guess" at the estimate and writing it as 4/10 gap ± 1/10 gap will cover all of the various differing estimate.

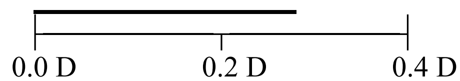
4) To convert our estimated fraction into a decimal value with units we multiply this quantity by value of the smallest gap. That is, (4/10 gap ± 1/10 gap) × 0.1 D/gap = 0.04 D ± 0.01 D.

5) Adding this estimate to the known value gives a final measurement of 0.14 D ± 0.01 D

In this instance, the measurement may be shortened to simply 0.14 D. A reported value, 0.14 D, conveys complete and concise information about our measurement of the length of the line. It would be interpreted, without additional information, by any scientist as indicating that the length was approximated to two figures and that the length is 0.14 D ± 0.01 D, or between 0.13 D and 0.15 D. (This is proper because that is our uncertainty. Remember, we said "give or take a tenth", and a tenth of 0.1 D is 0.01 D.) Therefore, to generalize, if a measurement is 1.234 D, then the uncertainty is ± 0.001 D; if a measurement is 1.23 D, then the uncertainty is ± 0.01D, and so on.

What should be done in those cases where the uncertainty in our estimate is greater than one unit in the last decimal place? How should such a measurement be reported? It should be reported so that the measurement **explicitly states the actual uncertainty**.

To illustrate this point, consider the following measurement (this is the same picture but a different scale):



Using our five-step process gives:

1) The numbers on the scale increase left to right and the value of the smallest gap is **0.2 D**.

2) The line is obviously longer than 0.2 D but less than 0.4 D and therefore we know the 0.2 D with absolute certainty. Once again, the line should be reported to two significant figures.

3) Note: as the “picture” didn’t change then our estimate is EXACTLY the same. Our estimate remains as 4/10 gap, give or take a tenth of a gap. I.e., 4/10 gap ± 1/10 gap.

4) To convert our estimated fraction into a decimal value with units we multiply this quantity by value of the smallest gap. Now this is, (4/10 gap ± 1/10 gap) × 0.2 D/gap = 0.08 D ± 0.02 D.

5) Adding this estimate to the known value gives a final measurement of 0.28 D ± 0.02 D.

This time however, it would be incorrect to report the value simply as 0.28 D, because this implies that the measurement is accurate to 0.01 D, which it is not. Our uncertainty is ± 0.02 D. (Remember our uncertainty was "give or take a tenth" of the distance between 0.2 D and 0.4 D; 1/10 of 0.2 D is 0.02 D.) Therefore, the value should be reported as 0.28 D ± 0.02 D. When expressed this way, it is clear that the uncertainty is 0.02 D, and not 0.01 D. Hence, once again we have conveyed complete and concise knowledge of a particular fact.

SIGNIFICANT FIGURES IN CALCULATIONS

There are rules to determine the number of significant figures that should be kept in your final answer which is the result of a calculation. However, you should retain one extra digit, in addition to the significant figures, in any intermediate calculation. (It is convenient to write this extra digit as a subscript to indicate that the digit is not significant.) These rules work well when the uncertainty in all of the numbers is only ± 1 in the last digit. For our purposes we will assume that this will always be the case.

At times, the result of a calculation contains more digits than are significant. Thus, the result must be "rounded off" to keep only the significant figures. In your final answer, the extra figures are eliminated by use of the following rules.

If the digit following the last significant figure is less than "5", then all of the unwanted digits are discarded and the last figure is kept unchanged (i.e. the number is truncated).

E.g., 1.8539 m (to 3 significant figures) is 1.85 m

If the digit following the last significant figure is more than "5", or is "5" with other nonzero numbers following it, then the last digit to be retained is increased by 1 ("rounded up") and the extra digits are dropped.

E.g., 3.23475 kg (to 4 significant figures) is 3.235 kg

and 45.43505 cm (to 4 significant figures) is 45.44 cm

If the digit following the last significant figure is a "5" and all other digits following the "5" are zeroes, then the number is either truncated or rounded up, so that the last significant digit is or becomes even.

E.g., 6.2655 mL (to 4 significant figures) is 6.266 mL

but 6.2665 ml (to 4 significant figures) is also 6.266 mL

The rounding rule for addition and subtraction is: the result of addition or subtraction is reported to the same number of decimal places as the number with the fewest decimal places.

E.g., 53.2×10^{-2} m + 5.32×10^{-3} m = 53.7×10^{-2} m

57.068 g + 44.3 g = 101.4 g

10.85 cm − 1.043 cm = 9.81 cm

The rounding rule for multiplication and division is: the result of multiplication or division is reported to the same number of significant figures as the number with the fewest significant figures (i.e., the least precise term) used in the calculation.

E.g., 152.06 N x 0.24 m = 36. N·m

Care and good judgement must also be exercised if, for example, you want the weight of 10 measured items, each weighing 5.25 g. If you add them, you get four significant figures; if you multiply 5.25 g x 10, you get three significant figures. Clearly three significant figures is more appropriate, since when you add the 10 numbers you also add the uncertainties. (That is, 5.25 g added 10 times is equal to 52.50 g ± 0.10 g or 52.5 g.)

SI UNITS

Not all numbers have a unit associated with them. If a number does have a unit associated with it, then the number must always be written with the corresponding unit. ALL measured numbers and counted numbers have units associated with them. Defined numbers may or may not have a unit associated with them. For example, the "2" in the formula: diameter = 2 x radius, is exact and has no unit. However, the "12" in 12 inches per foot has the units "inches per foot."

Worldwide, the units used in the sciences are not those of the English system of weights and measures (for example, pounds and inches). Instead, the units used belong to Le System International d'Unites, which is abbreviated as the SI Units. The only technical field which still uses the English system is the field of Engineering in the United States. All other countries have abandoned the English system completely. The clear advantages of SI Units will become apparent as we discuss them.

There are only seven "SI Base Units" in this system. Each one represents one of the seven basic properties that can be independently measured. We shall present the definitions for each of these units as we need them. These units are listed below, along with the property they are used to measure and the official symbol or abbreviation.

PROPERTY	SI UNIT	SYMBOL
length	**meter**	m
mass	**kilogram**	kg
time	**second**	s
temperature	**kelvin**	K
amount of substance	**mole**	mol
electric current	**ampere**	A
luminous intensity	**candela**	cd

All conversion factors between different magnitudes of the same property use the SI unit and some multiple of ten. The exact multiple selected is indicated by the choice of the prefix attached to the base unit. (See the complete **Table of Prefixes** at the end of this handout.)

Some useful combinations of the base units which do not give a new SI Unit are:

area	m^2
velocity	m/s
acceleration	$(m/s)/s = m/s^2$
momentum (= mass x vel.)	kg·m/s.

Some other combinations of the base units give SI Derived Units

PROPERTY	SI UNIT	SYMBOL	UNIT COMBINATION
Volume	**liter**	L	$(10^{-1}\text{m})^3$ or $(\text{dm})^3$
Force = mass x accel. (= weight)	**newton**	N	$\text{kg}\cdot\text{m/s}^2$
Energy = force x dist. (= work)	**joule**	J	$\text{kg}\cdot\text{m}^2/\text{s}^2 = \text{N}\cdot\text{m}$
Power = energy/time	**watt**	W	$\text{kg}\cdot\text{m}^2/\text{s}^3 = \text{J/s}$
Pressure = force/area (= energy/volume)	**pascal**	Pa	$\text{kg/m}\cdot\text{s}^2$
Electrical charge	**coulombs**	C	$\text{A}\cdot\text{s}$
Electrical potential	**volt**	V	W/A = J/C

All of these units (and properties) are defined by the base units from which they are derived.

The liter is an interesting case, in that it is a length cubed (a volume), but not equal to the base unit (meter) cubed but is instead defined as a thousandth of a cubic meter, i.e. 1 liter = 1/1000 m^3.

Table of Prefixes

PREFIX	Symbol	FACTOR		
exa–	E	10^{18}	or	1 000 000 000 000 000 000
peta–	P	10^{15}	or	1 000 000 000 000 000
tera–	T	10^{12}	or	1 000 000 000 000
giga–	G	10^{9}	or	1 000 000 000
mega–	M	10^{6}	or	1 000 000
kilo–	k	10^{3}	or	1 000
hecto–	h	10^{2}	or	100
deca–	da	10^{1}	or	10
-------the base unit‡--------				
deci–	d	10^{-1}	or	0.1
centi–	c	10^{-2}	or	0.01
milli–	m	10^{-3}	or	0.001
micro–	μ*	10^{-6}	or	0.000 001
nano–	n	10^{-9}	or	0.000 000 001
pico–	p	10^{-12}	or	0.000 000 000 001
femto–	f	10^{-15}	or	0.000 000 000 000 001
atto–	a	10^{-18}	or	0.000 000 000 000 000 001

‡except for the mass, where **kilo**gram is the base unit.

Never combine prefixes. For example, even though kilogram is the base unit, 1000. kilograms can be expressed as 1000. kg or 1.000 Mg, but not 1.000 kkg.

*The symbol for "micro-" is the Greek letter, μ (pronounced "mu").

You should definitely know the prefixes: mega-, kilo-, deci-, centi-, milli-, micro-, nano-, and pico-.

Numbers and the SI Units : Homework

1. How many significant figures are there in each measurement?

   a) 52 mm b) 1.0023 g c) 12.50 s

   d) 0.0113 cm e) 1.5×10^{-2} mL

2. Round off the following measurements.

   a) 1.234501 g (to 4 Sig Figs) =

   b) 0.01234 g (to 3 decimal places) =

   c) 0.05678 g (to 3 Sig Figs) =

   d) 1234567 km (to 4 Sig Figs) =

3. Report the result of each calculation to the correct number of significant figures.

   a) 8.59 g + 1.61 g =

   b) 0.123 cm + 5.56 cm + 3.3 cm =

   c) 1.250 mL + 1.25 mL − 2.50 mL =

   d) 1.23 cm + 1.23 mm =

4. Report the result of each calculation to the correct number of significant figures.

   a) 1.34 cm x 3.59 cm x 0.62 cm =

   b) 25 N x 2.63 m =

   c) $\dfrac{52.4\text{ N}}{6.256\text{ m} \times 3.4\text{ m}} =$

   d) $1.25\text{ g} \div (0.500\text{ cm})^3 =$

5. Make the following unit conversions.

   a) 0.25 m = ___________ cm

   b) 372 mL = ___________ L

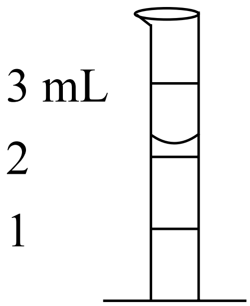
   c) 0.25 ms = ___________ s

   d) 125 pm = ___________ nm

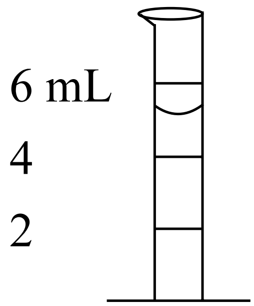
   e) 125 cm^3 = ___________ m^3

   f) 7.8×10^3 g/L = ___________ kg/ml

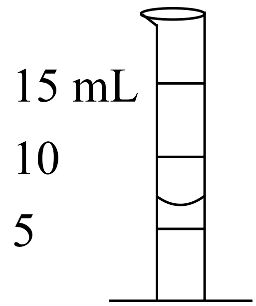
6. Make the following measurements.



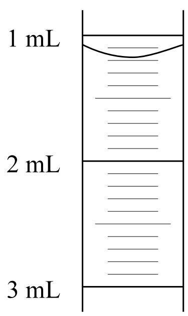
a.________________



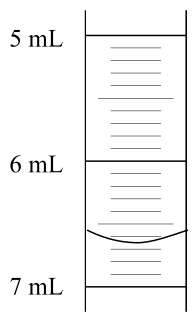
b.________________



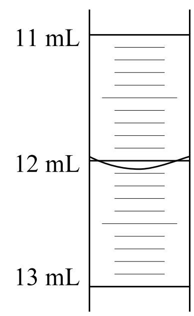
c.________________



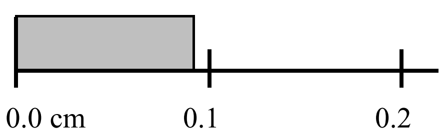
d.________________



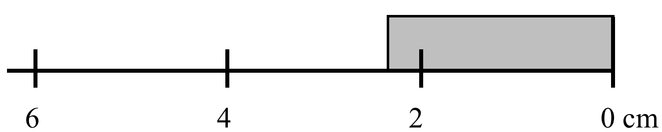
e.________________



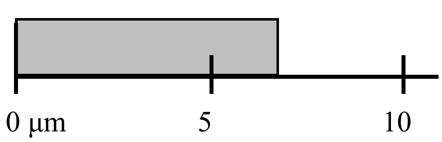
f.________________



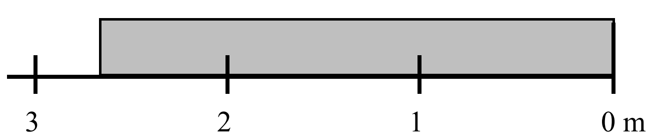
g.____________________



h.____________________



i.____________________



j.____________________

Numbers and the SI Units : Homework Answer Key

1. As stated on page 2 of the handout:

   To determine the number of significant figures in a given measured number, begin at the left-hand side of the number and move toward the right. The first non-zero digit that you encounter and all digits to the right of it are significant.

   Thus, the correct number of significant figures in the measurements are:

   a) 2 b) 5 c) 4 d) 3 e) 2

2. Following the procedures outlined on page 4 of the handout, the measurements rounded off to the stated numbers of significant figures are:

   a) 1.235 g (The digit following the last significant figure **is** a "5" followed by other nonzero numbers.)
   b) 0.012 g (To 3 decimal places not significant figures--as in rounding off after addition or subtraction.)
   c) 0.0568 g
   d) 1.235×10^6 km (You cannot express a LARGE seven digit number to four significant figures without using scientific notation)

3. The result of addition or subtraction is reported to the same number of decimal places as the number with the fewest decimal places.

   Thus answers to the correct number of significant figures are:

   a) 10.20 g

   b) 9.0 cm

   c) 0.00 mL (surprise!)

   d) 13.5 mm or 1.35 cm

4. The result of multiplication or division is reported to the same number of significant figures as the number with the fewest significant figures (i.e., the least precise term) used in the calculation.

   Thus answers to the correct number of significant figures are:

   a) 3.0 cm^3

   b) 66 N·m

   c) 2.5 N/m^2

   d) 10.0 g/cm^3

5. a) $0.25 \text{ m} \times \dfrac{1 \text{ cm}}{1\times10^{-2} \text{ m}} = 25 \text{ cm}$

b) $372 \text{ mL} \times \dfrac{1\times10^{-3} \text{ L}}{1 \text{ mL}} = 0.372 \text{ L}$

c) $0.25 \text{ ms} \times \dfrac{1\times10^{-3} \text{ s}}{1 \text{ ms}} = 2.5\times10^{-4} \text{ s}$

d) $125 \text{ pm} \times \dfrac{1\times10^{-12} \text{ m}}{1 \text{ pm}} \times \dfrac{1 \text{ nm}}{1\times10^{-9} \text{ m}} = 1.25\times10^{-1} \text{ nm} = 0.125 \text{ nm}$

e) $125 \text{ cm}^3 \times \left(\dfrac{1\times10^{-2} \text{ m}}{1 \text{ cm}}\right)^3 = 1.25\times10^{-4} \text{ m}^3$

f) $7.8\times10^3 \text{ g/L} \times \dfrac{1 \text{ kg}}{1\times10^3 \text{ g}} \times \dfrac{1\times10^{-3} \text{ L}}{1 \text{ mL}} = 7.8\times10^{-3} \text{ kg/mL}$

6. Following the procedures outlined on page 3 of the handout the proper measurements are:
NOTE: The quantities in parentheses are NOT required.

a) 2.2 mL (± 0.1 mL)
b) 5.2 mL ± 0.2 mL
c) 7.0 mL ± 0.5 mL
d) 1.18 mL (± 0.01 mL)
e) 6.64 mL (± 0.01 mL)
f) 12.07 mL (± 0.01 mL)
g) 0.09 cm (± 0.01 cm)
h) 2.4 cm ± 0.2 cm
i) 7.0 µm ± 0.5 µm
j) 2.7 m (± 0.1 m)