## Cycloids

## Definition

A cycloid is the curve traced by a point on the edge of a circle as it rotates along a straight line without slip.


## History

Discovered in 1599 by Galileo, the cycloid has found its way into many applications since then. It was used by many mathematicians such as Roberval, Wren, Huygens, and Bernoulli to attempt to find the area by weighing pieces of metal cut into the shape of a cycloid, designs for early gear teeth technologies, and was the key part of solving Bernoulli's problem.

## Equations

$$
\begin{aligned}
& x=r t-r \sin (t) \quad y=r-r \cos (t) \\
& x=\operatorname{rarccos}\left(\frac{r-y}{r}\right)-\sqrt{2 r y-y^{2}} \\
& \text { Arc length }=8 r \text { for one period }
\end{aligned}
$$

$$
\text { Area under cycloid }=3 \pi r^{2}
$$



The involute properties of the Cycloid. Demonstration of how the involute of Cycloid is same as Cycloid.

## Related Curves

Cycloidal Pendulum Trochoid
Hypotrochoid
Epitrochoid
Hypocycloid
Epicycloid

## Applications

## Rotary Pumps

Design of the gear tooth profiles.
Kinematics
Mechanical Transmissions
Thermal Engines
Architecture
Pendulum Clock

## References

https://mathworld.wolfram.com/Cycloid.html
https://www.irishtimes.com/news/science/the-curved-history-of-cycloids-from-galileo-to-cycle-gears1.2347025

