Bézier Curve

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ABSTRACT

How can we create smooth curves on a graph? Using a Bézier curve!

With at least three points and a concept called linear interpolation, a parametric curve known as a Bézier Curve can be used to create smooth lines on a graph.

INTRODUCTION

Computer graphic designers, engineers, animators and many more need to use smooth lines within their work. The Bézier Curve makes this task much more feasible.

The parametric curve consists of an origin point, any number of control points and an end point.

Linear interpolation uses of a range of numbers and a normalized value, then extract a number that corresponds to the normalized value.

Because it is a parametric curve, the points are (x(t), y(t)). t will be the normalization value between 0 and 1.

RESEARCH

- A Bézier curve consists of at least two points, a starting point and an ending point, and utilizes linear interpolations between the points.
- Using the generalized formula:

$$B(t) = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!}(1-t)^{n-i}P_i$$

where
t is a value (between 0 and 1) that determines a unique point on the curve
P_i is the respected control point
n is the degree and is dependent on the number of points (n=1: linear, n=2: quadratic, n=3: cubic)

- A higher n yields a complicated shape which provides the Bézier curve with a multitude of applications, ranging from physical design to virtual graphics.

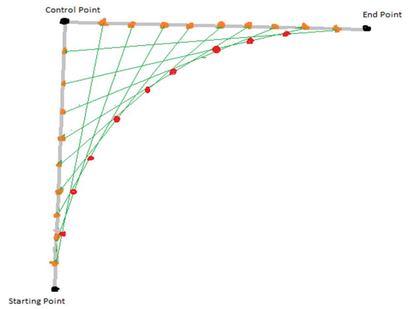

Figure 1: A quadratic interpolation with 10 interpolations

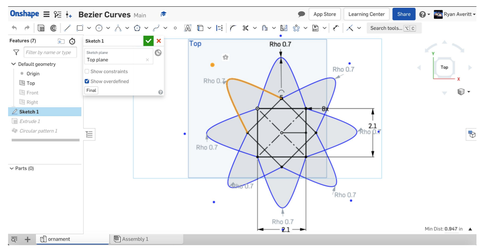

Figure 2: CAD Application

SUMMARY

The Bézier curve is a smooth parametric curve used in computer graphics, animation, modeling, CAD, etc.

The curve is formed from linear interpolation; more interpolation between points means a smoother curve.

There are at minimum three points. Their connection is not what makes the smooth curve - the connection between the interpolations does.

Without the Bézier curve, many technological advancements may not have been accomplished.

REFERENCES

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