Limaçon Curves

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History

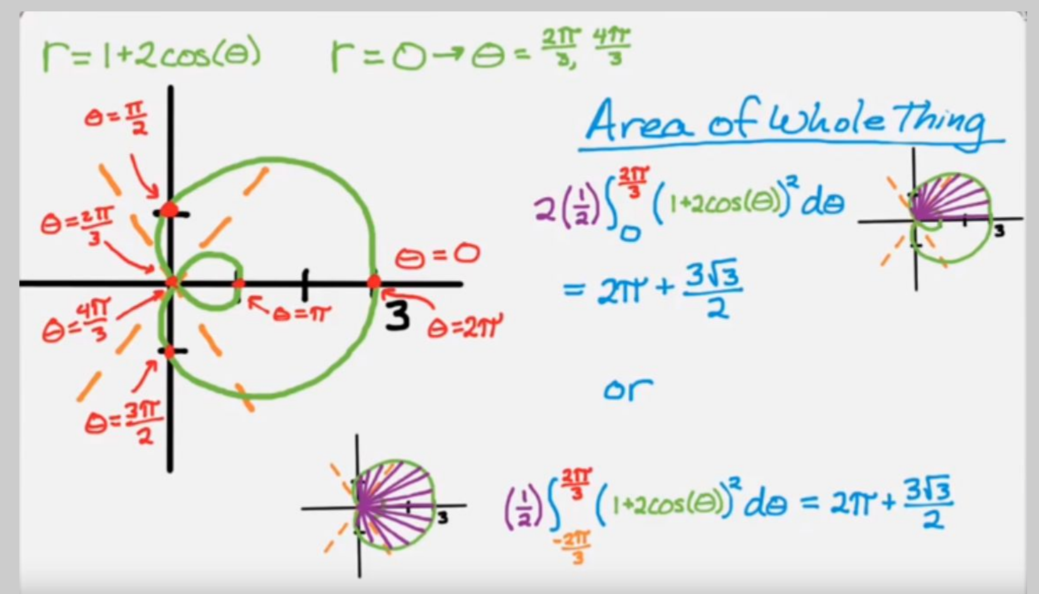
- First Investigated by Albrecht Durer
- Researched further by Etienne Pascal (has famous father)
- Named Limacon in 1650 by Gilles-Personne Roberval
- Limacon comes from the Latin limax, meaning snail

What is a Limacon?

- A limacon is a polar curve formed by rotating a circle around the border of a fixed circle of equal radius.
- The basic equation, in polar coordinates of a limacon are:
  r= a±bcos(θ)   r= a±bsin(θ)
- Different shapes related to the limacon can be formed through various inputs and ratios of a and b
  - Inner loop limacon
  - One loop limacon
  - Cardioid
- Other polar curves similar to Limacons include
  - Rose curves
  - Archimedes' spiral
  - Lemniscates

The Math of the Limacon

Area of polar curves:



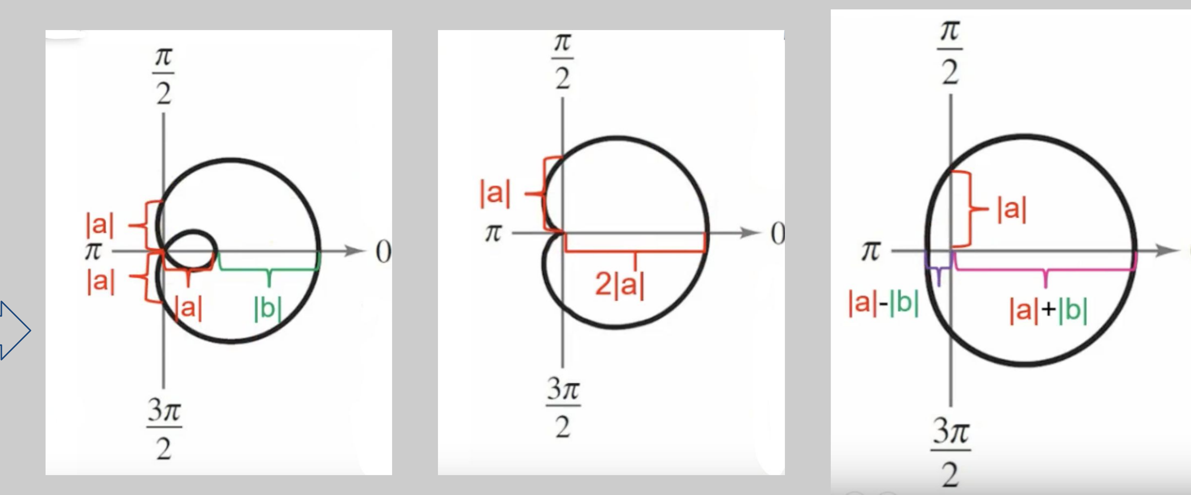
Formula for area of polar curves

$$A=\frac{1}{2}\int_{\theta_1}^{\theta_2} r^2\,d\theta$$

r= a±bcos(θ)
- When Cos is positive, graph lays on positive x-axis,
- When Cos is negative, graho lays on negative x-axis

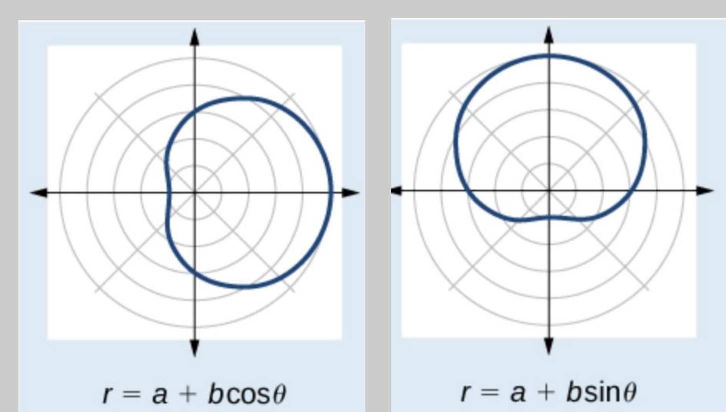
r= a±bsin(θ)
- When Sin is positive, graph lies on positive y-axis.
- When Sin is negative, graph lays on negative y-axis

Graphing Limaçon Curves



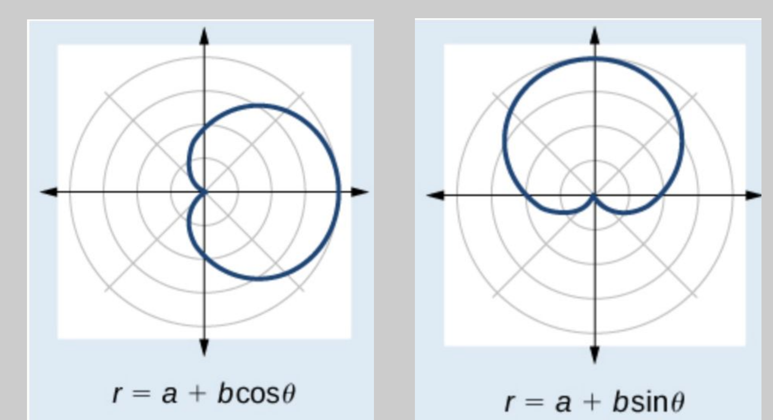
Inner loop   Cardioid   No Loop

One Loop Limacon



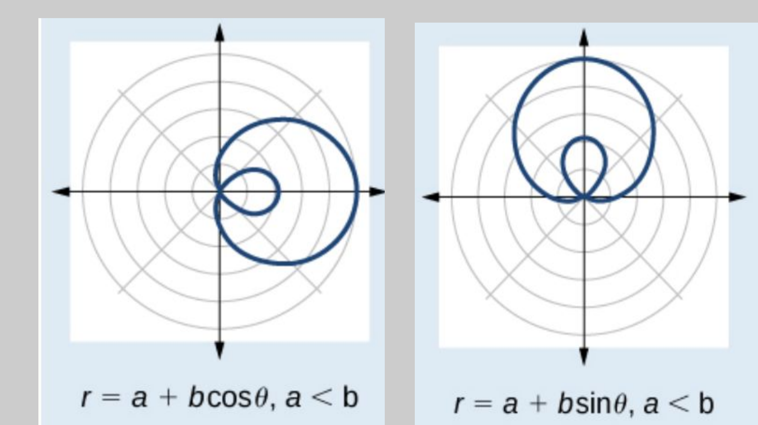
Dimple: when 1< a/b < 2
Convex: when a/b ≥ 2

Cardioid

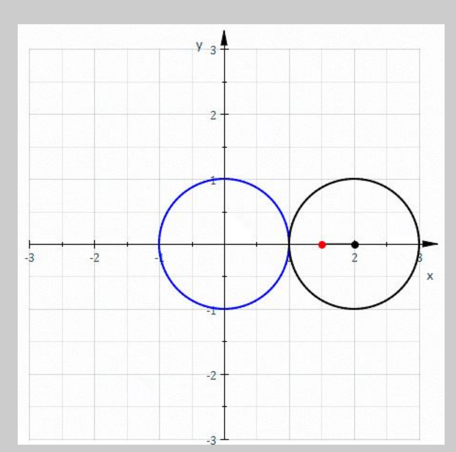


When a>0, b>0, and a/b = 1

Inner loop Limacon

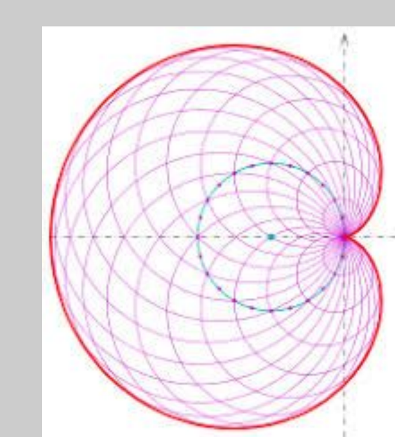


When a/b< 1



path of a point fixed to a circle when that circle rolls around the outside of a circle of equal radius

Albrecht Durer, the first person to research limacon curves



The cardioid, a special case of a limacon r=b+acos(Θ) where b=a

Tips and Tricks

r= a±bcos(θ)   r= a±bsin(θ)

Limacons containing cosine are symmetrical to x-axis.

Limacons containing sine are symmetrical to y-axis.

To find intercepts on the axis to which the limacon is symmetrical to, you can use the origin (0,0) and (a+b, 0) or (0, a+b) respectively.

When finding the area of a limacon (or polar curves in general) make use of its symmetry as much as you can so we calculate half the curve then multiply it by 2 to find the total area

References

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